Taylor and Maclaurin Series

1 Definition

The **Taylor Series** for a function $f(x)$ that is infinitely differentiable at a point a is

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots \tag{1}$$

which can be written as

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n \tag{2}$$

The **Maclaurin Series** is a Taylor Series where $a = 0$.

2 Maclaurin Series of Common Functions

All of the following equations are valid for complex arguments x.

2.1 Exponential Function

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{3}$$

2.2 Natural Logarithm

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} \qquad |x| < 1 \tag{4}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} \qquad |x| < 1 \tag{5}$$

2.3 Trigonometric Functions

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \tag{6}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \tag{7}$$

2.4 Geometric Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad |x| < 1 \tag{8}$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1} \qquad |x| < 1 \tag{9}$$

$$\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} n x^n \qquad |x| < 1 \tag{10}$$