Impulse Function

1 Definition

The **Impulse Function**, also known as the **Dirac Delta Function**, is defined as follows:

$$\delta(x) = \begin{cases} +\infty & x = 0 \\ 0 & x \neq 0 \end{cases} \tag{1}$$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \tag{2}$$

The impulse can also be thought of as a zero-centred normal distribution with zero standard deviation.

$$\delta(x) = \lim_{\sigma \to 0} \frac{1}{\sigma\sqrt{\pi}} e^{\frac{-x^2}{\sigma^2}} \tag{3}$$

2 Properties

2.1 Convolution

$$f(x) \star \delta(x) = f(x) \tag{4}$$

2.2 Sifting

$$\int_{-\infty}^{\infty} \delta(x - x_0) f(x)dx = f(x_0) \tag{5}$$